AIAA JOURNAL
Vol. 31, No. 11, November 1993

Uncertainty Estimates for Pressure Sensitive Paint Measurements

Miklos Sajben*
McDonnell Douglas Aerospace, St. Louis, Missouri 63166

A recently introduced surface pressure measurement technique for aerodynamic applications uses pressure sensitive luminescent coatings. The primary measurable is luminescent light intensity, but the determination of pressure also requires the measurement of other variables, such as a reference light intensity and surface temperatures. Each of the primary measurables is associated with an error that contributes to the uncertainty of the pressure computed from all inputs. This paper investigates these contributions to the resultant uncertainty, both in fundamental terms and in terms of commonly used wind tunnel parameters. The uncertainty is sharply dependent on both freestream and local flow conditions, such that a simple global characterization of error magnitude does not seem to be practical. Spatial surface temperature variations and temperature differences between reference and run conditions may significantly affect the results.

Introduction

A SURFACE pressure measurement technique using pressure sensitive paints (PSP) is currently under development for aerodynamic tests in wind tunnels and other flow facilities.[1–6] The method offers the promise of extensive surface pressure information at low cost and in a short time. The method is highly unconventional, and traditional approaches to estimating the overall precision and accuracy are not applicable. The present paper addresses one aspect of uncertainty estimation: how errors in the various directly measured physical quantities affect the precision of the pressure data computed from them. An understanding of the relative magnitudes of these effects is helpful in determining the best allocation of resources in relevant development efforts. Attention is focused on the ratio method, in which the pressure is computed from the ratio of light intensities determined with and without flow over the surface of interest.

Description of the Method

Detailed descriptions of the pressure sensitive paint technique are available in the literature[2–5] and only a brief summary is given here.

The technique is based on a class of luminescent materials for which the luminescence is diminished (quenched) by the presence of oxygen. Using a fixed composition gas (air), the oxygen concentration is linearly proportional to the gas pressure, and luminescence intensity becomes a measure of the surface pressure. The luminescent material is dispersed in an oxygen-permeable binder material that provides adhesion to the surface. The model is primed and coated with the luminophore/binder mixture, usually after installation in the tunnel. During the measurement, the model is illuminated by light in a wavelength range appropriate for the particular compound. Luminescence occurs at a longer wavelength that is also a characteristic of the paint used. The luminescent image of the model is recorded, usually by a digital video camera equipped with a narrow-band filter that matches the emission wavelength range. This image is the input to a data reduction procedure, the fundamentals of which are described later.

We assume that the effect of pressure on the luminescence (emitted light) intensity is well described by the Stern-Volmer relation.[3–6]

$$I_o/I = 1 + Kp \tag{1}$$

where I is the luminescent light intensity at gas pressure p; I_o is the luminescent light intensity in the absence of oxygen (unquenched); K is a quantity characteristic of the paint layer, called the Stern-Volmer constant and is, in this analysis, assumed to be a known function of the surface temperature; and p is the gas pressure over the surface.

Equation (1) is not necessarily valid for all paint formulations: many may display more complex dependencies on pressure and temperature. The uncertainties associated with such paints are beyond the scope of this article.

The emitted light intensity is proportional to the incident light intensity, and we can write:

$$I_o = QI_e \tag{2}$$

where I_e is the excitation (incident) illumination intensity, and Q is the "efficiency" of the paint layer, assumed to be a known function of surface temperature.

The incident illumination is generally directional, depending on the location of the light source(s). In contrast, the luminescence is diffuse (with the possible exception of very shallow viewing angles, which will not be considered here).

Combination of Eqs. (1) and (2) yields

$$I = QI_e/(1 + Kp) \tag{3}$$

Equation (3) is the basis of the current analysis. It contains two temperature-dependent material constants K and Q. Because we are assuming that these two functions are known from an appropriate calibration, the knowledge of surface temperature is tantamount to knowing both K and Q. We also assume that spatial and temporal variations in the deposited paint properties are negligible, i.e., K and Q are not dependent on location over the model surface, nor do they change in time (aging or photo-bleaching does not occur).

Equation (3) is viewed as containing four quantities: surface pressure, surface temperature (which implicitly defines both K and Q), and the incident and emitted light intensities. Measurement of the luminescent intensity is clearly not sufficient to determine the desired pressure: two more unknowns are present.

Received Aug. 7, 1992; revision received April 26, 1993; accepted for publication April 27, 1993.

*McDonnell Douglas Fellow, Propulsion and Thermodynamics, M/S 111-1041; currently Professor, University of Cincinnati, Department of Aerospace Engineering and Engineering Mechanics, ML 70, Cincinnati, OH 45221. Fellow AIAA.

A convenient procedure exists, called the ratio method, that can eliminate the need to determine the excitation light intensity. This approach requires an additional luminescent image of the same object, with the same illumination, but in the absence of flow. No flow assures a constant pressure at all points of the surface. The image taken this way is called the reference image ("ref"), to differentiate from the image taken with flow, which is referred to as the "run" image. Designating the ref condition with an asterisk superscript, Eq. (3) is specialized as

$$I^* = \frac{Q^* I_e^*}{1 + K^* p^*} \tag{4}$$

where we used the notation $Q^* = Q(T^*)$ and $K^* = K(T^*)$. If the surface temperatures are the same in both the ref and the run cases ($T = T^*$), and if the excitation intensities are the same ($I_e = I_e^*$), then division of Eq. (3) by Eq. (4) yields:

$$\frac{I}{I^*} = \frac{1 + Kp^*}{1 + Kp} \tag{5}$$

The luminescent intensities are measured in both cases. The reference pressure is also readily measured, leaving only K and p as unknowns. Under the assumptions made so far, K may be determined from comparison with pressure measured by a single pressure tap, or from a single surface temperature measurement. Once K is determined, the unknown pressure p is readily evaluated from Eq. (5).

There are other approaches to handling the undesirable unknowns in Eq. (3) but they are not considered here.

Estimating Uncertainties

The assumptions implicit in the ratio method are likely to be satisfied only approximately. Furthermore, the measurements of light intensities and the reference pressure are also subject to certain tolerances. It is therefore of interest to inquire about the magnitude of the uncertainty in pressure that results from some specified uncertainty in any of the input quantities.

If the ref and run illuminations and temperatures are not exactly the same, then the division of Eq. (3) by Eq. (4) yields the following general expression for pressure:

$$p = \frac{I^*}{I}\frac{Q}{Q^*}\frac{I_e}{I_e^*}\frac{(1 + K^* p^*)}{K} - \frac{1}{K} \tag{6}$$

This equation gives the (run) pressure as the function of seven variables: I, I^*, I_e, I_e^*, T, T^*, and p^*. Uncertainties in any of these give rise to a corresponding uncertainty in p. The respective errors are estimated by assuming that the errors are small, and the factors of proportionality are given by the partial derivatives of Eq. (6) with respect to the input variables. For example, the pressure error due to error in one of the input variables is given by

$$\mathrm{d}p_n = \frac{\partial p}{\partial x_n}\,\mathrm{d}x_n \tag{7}$$

where x_n is any one of the input quantities I, I^*, I_e, etc.

It is convenient and often more meaningful to look at the fractional change in each property, by rewriting Eq. (7) as follows:

$$\frac{\mathrm{d}p_n}{p} = \frac{x_n}{p}\frac{\partial p}{\partial x_n}\frac{\mathrm{d}x_n}{x_n} = \phi_n \frac{\mathrm{d}x_n}{x_n} \tag{8}$$

The coefficients ϕ_n are called the "influence coefficients." The differential of the independent variable may represent uncertainty associated with any arbitrarily chosen level of confidence, i.e., with any number of standard deviations. The uncertainty computed for the dependent variable corresponds to the same confidence level.

The calculation of the temperature coefficients requires the evaluation of the logarithmic derivatives of K and Q with respect to temperature. For this purpose, the empirically determined temperature dependencies are approximated by power law functions

$$K(T) = a_K T^{\nu_K} \tag{9a}$$

$$Q(T) = a_Q T^{\nu_Q} \tag{9b}$$

where a_K and a_Q are constants and T is the absolute temperature. Absolute temperature is used as an independent variable to avoid exponents that depend on the choice of the temperature scale. Using the expressions of Eq. (9), the logarithmic derivatives are found to be constants, equal to the respective exponents. We have

$$\frac{\mathrm{d}K/K}{\mathrm{d}T/T} = \frac{\mathrm{d}\,\ell_n K}{\mathrm{d}\,\ell_n T} = \nu_K \tag{10a}$$

$$\frac{\mathrm{d}Q/Q}{\mathrm{d}T/T} = \frac{\mathrm{d}\,\ell_n Q}{\mathrm{d}\,\ell_n T} = \nu_Q \tag{10b}$$

This form of approximation was found to be adequate for a modest temperature range (±15 K), leads to simple analytical expressions, and is used throughout this paper.

The seven influence coefficients have been evaluated, expressed as functions of Kp and K^*p^*, and are summarized in Table 1.

When evaluated for a particular paint and flow (i.e., for a given Kp, K^*p^*, ν_K, ν_Q combination), the influence coefficients describe the importance of the various input quantities and may be used to determine the precision required in each, for a given desired accuracy in p. Figure 1 shows the example of ϕ_T, i.e., the influence coefficient for the run temperature, using data for an early paint developed by McDonnell Douglas Laboratories (designated MDRL-PF1, $\nu_K = 3.1$, $\nu_Q = -2.6$). The plot indicates that run temperature is a critical factor at low pressure levels. The influence coefficient for ref temperature (not illustrated) has a comparable dependence on Kp.

The form of ϕ_T given in Table 1 suggests how the influence of temperature uncertainties on the pressure measurement might be reduced. By setting the expression for ϕ_T equal to zero, the following relation is obtained between paint properties and the pressure:

$$\left(\frac{\nu_K}{\nu_Q} - 1\right)K = \frac{1}{p} \tag{11}$$

Table 1 Influence coefficients

x_n	ϕ_n
I	$-y$
I^*	y
I_e	y
I_e^*	$-y$
T	$y\left(\nu_Q - \frac{\nu_K}{y}\right)$
T^*	$-y\left(\nu_Q - \frac{\nu_K}{y^*}\right)$
p^*	$\frac{y}{y^*}$

$$\left\{\begin{array}{l} \dfrac{\mathrm{d}p_n}{p} = \phi_n\left(\dfrac{\mathrm{d}x_n}{x_n}\right) \\ y = 1 + \dfrac{1}{Kp}, \quad y^* = 1 + \dfrac{1}{K^* p^*} \end{array}\right.$$

If a paint could be developed that satisfies Eq. (11), then the influence coefficient would be zero and temperature uncertainties would be irrelevant. Because p varies over surfaces of interest, exact cancellation is possible only at selected points. But even an approximate conformity to Eq. (11) could substantially lower temperature-related errors.

If the errors of the input quantities are independent, then the resultant pressure error from all causes is estimated as the square root of the sum of the squares of individual errors. Table 2 shows a sample estimate for a particular paint, for a hypothetical pressure measurement at nearly atmospheric conditions, using reasonable assumptions for each input uncertainty. The results show that the assumed temperature uncertainty (dT = 1.5°C) accounts for 93% of the pressure uncertainty and thus dominates the measurement. To bring the consequences of the temperature uncertainty to a level comparable to the rest of the inputs, the fractional temperature uncertainty would have to be reduced to 0.001, corresponding to about $dT = 0.3$°C.

The precision of any of the input data can be improved by repeating the respective measurement N times and averaging the results. This procedure will reduce the contribution to the resultant pressure error (the respective entry in the third column of Table 2) by a factor of 1/N.

Adaptation to Wind Tunnel Practice

The influence coefficients of Table 1 characterize the ratio method version of PSP, without reference to the special circumstances of wind tunnel use. The conditions of practical wind tunnel testing require some additional considerations.

A very important fundamental feature of PSP is that the method is sensitive to absolute pressure, in contrast to conventional practice that focuses on gauge pressures. In a typical wind tunnel test, differential pressure transducers are used to measure deviations from the freestream pressure. Lift force is obtained by integrating the pressure difference between the bottom and top sides of the model. If the pressure difference is small compared with the absolute pressure level, then the determination of lift from absolute pressure measurements amounts to taking small differences of large numbers, with a corresponding decrease of accuracy. Consequently, PSP is more appropriate for higher Mach numbers, where dynamic pressures are high compared with the static pressure.

The practice of measuring differential pressures is reflected by the common use of dimensionless pressure coefficients defined as

$$C_p = \frac{p - p_\infty}{1/2\,\rho_\infty U_\infty^2} \tag{12}$$

where p_∞, ρ_∞, and u_∞ are the pressure, density, and velocity, respectively, in the undisturbed freestream approaching the model.

Table 2 Sample estimate of run-pressure uncertainty[a]

Input quantity	Assumed input uncertainty	Contributions	
x_n	$\frac{dx_n}{x_n} \times 10^2$	$\left(\phi_n \frac{dx_n}{x_n}\right)^2 \times 10^4$	Percentage of total
I	0.5	0.373	1.7
I^*	0.5	0.373	1.7
I_e	0.5	0.373	1.7
I_e^*	0.5	0.373	1.7
T	0.5	9.851	45.5
T^*	0.5	10.313	47.5
p^*	0.5	0.003	0.2
Totals:		21.659	100.0
Resultant pressure uncertainty; dp/p = 0.0465 (4.7%)			

[a]Assumptions used: Paint: $K = K^* = 0.06$ kPa^{-1}, $\nu_K = 3.1$, $\nu_Q = -2.6$. Flow: p = 75 kPa, p^* = 100 kPa, $T = T^*$ = 300 K.

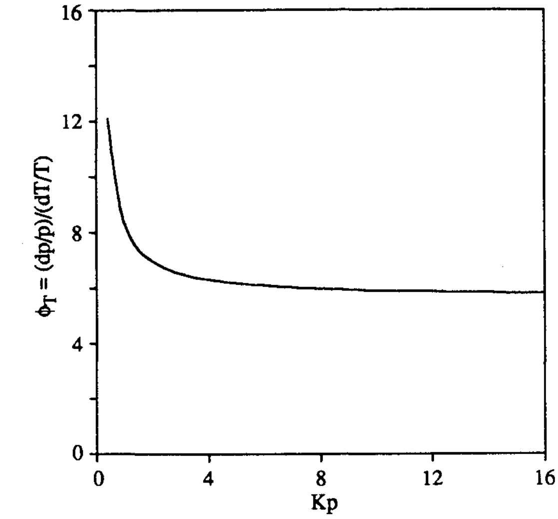

Fig. 1 Influence coefficient (fractional pressure change per fractional temperature change) for an early pressure sensitive paint.

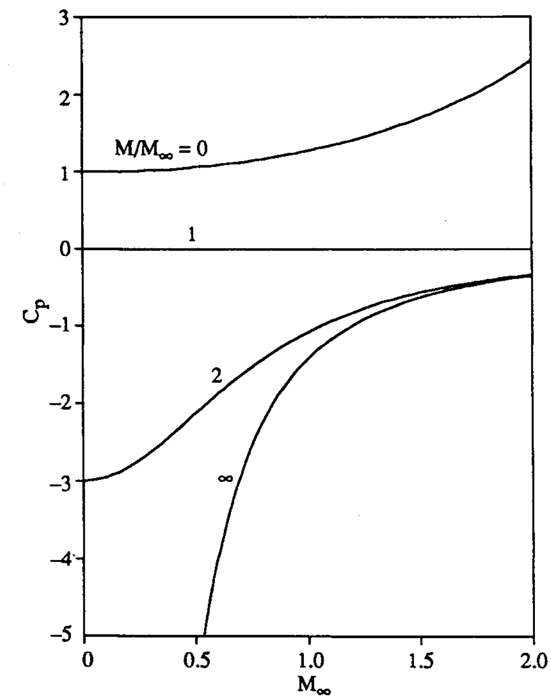

Fig. 2 Pressure coefficient as a function of freestream Mach number and local-to-freestream Mach number ratio. Parameter values (0, 1, 2, ∞) correspond to special conditions on the surface.

An alternate expression for C_p can be given in terms of the freestream Mach number of the tunnel as follows:

$$C_p = \frac{(p/p_\infty - 1)}{(\gamma/2)\,M_\infty^2} \tag{13}$$

where γ is the ratio of specific heats ($\gamma = 1.4$ for air). C_p values are of order unity for all absolute pressure levels and tunnel Mach numbers.

It follows from the previous equation that the accuracy of aerodynamic force information is most closely related to the accuracy of information available in the form of pressure coefficients. It is therefore appropriate to investigate the error in C_p, as opposed to the error in p itself. The connection between the two is given by taking the differential of Eq. (13):

$$dp = (\gamma/2) M_\infty^2 p_\infty dC_p \tag{14}$$

The local surface pressure on a wind tunnel model depends in part on the wind tunnel conditions and in part on the local Mach number (or local C_p). These influences are directly reflected by the choice of the following set of three dimensionless parameters.

Wind tunnel conditions are described by $Kp_{t\infty}$ as the tunnel total (stagnation) pressure, normalized by the Stern-Volmer constant K, and M_∞ as the freestream Mach number.

The local conditions on the model surface are characterized by M/M_∞ as the ratio of local Mach number M to the freestream Mach number.

M/M_∞ has three significant values. $M/M_\infty = 0$ designates stagnation points where C_p reaches its maximum value. $M/M_\infty = 1$ corresponds to locations where the local pressure is equal to freestream pressure, i.e., $C_p = 0$. A third significant value could be chosen to correspond to the minimum value of C_p, but this implies that the minimum of C_p would occur at an infinite local Mach number and is never reached in any practical situation. As a pragmatic alternative we shall use $M/M_\infty = 2$ as the low end of the practical range. The local pressure coefficient C_p is defined by M and M/M_∞ jointly; the special values chosen here are illustrated in Fig. 2.

The dimensionless pressure Kp is connected to the tunnel parameters through the relation

$$Kp = \frac{Kp_{t\infty}}{[1 + (M/M_\infty)^2 (\gamma - 1)/2M_\infty^2]^{\gamma/(\gamma-1)}} \tag{15}$$

where we used the well-known relation between static and total pressures

$$\frac{p_t}{p} = \left[1 + \frac{\gamma - 1}{2} M^2\right]^{\gamma/(\gamma-1)} \tag{16}$$

Using Eqs. (14) and (15), modified versions of the influence coefficients can be easily obtained in the form

$$dC_p = \psi_n \frac{dx_n}{x_n} \tag{17}$$

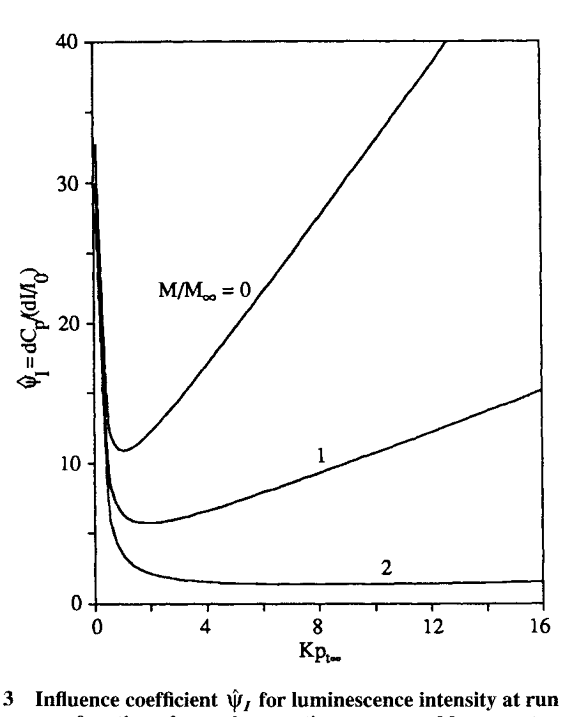

Fig. 3 Influence coefficient $\hat{\psi}_I$ for luminescence intensity at run conditions, as a function of tunnel stagnation pressure: M_∞ = const = 1.0.

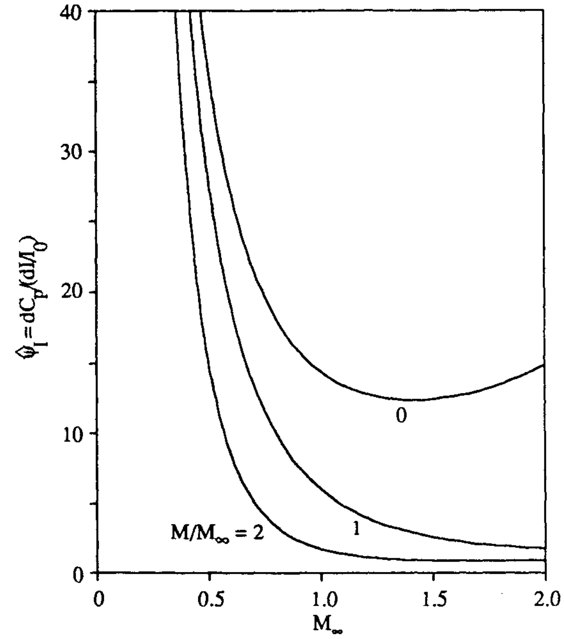

Fig. 4 Influence coefficient $\hat{\psi}_I$ for luminescence intensity at run conditions as a function of freestream Mach number: $Kp_{t\infty}$ = const = 3.0.

where ψ_n is the modified influence coefficient for input quantity x_n.

To illustrate the use of modified influence coefficients, we shall examine their behavior as functions of tunnel and local conditions for two selected input quantities: the luminescent light intensity I and the temperature T.

Influence Coefficient for I

From Table 1

$$\phi_I = \frac{dp/p}{dI/I} = -\left(1 + \frac{1}{Kp}\right) \tag{18}$$

Eliminating dp using Eq. (14) yields

$$\frac{dC_p}{dI/I} = -\frac{2}{\gamma M_\infty^2}\left(\frac{p}{p_\infty}\right)\left(1 + \frac{1}{Kp}\right) \tag{19}$$

One additional modification is made on the basis of related experience that in light intensity measurements the error tends to be only a weak function of the intensity itself. It follows that for luminescence measurements, a fixed absolute error is a more realistic model than a fixed relative error. Accordingly, we shall replace dI/I with

$$\frac{dI}{I} = \frac{dI}{I_o}\left(\frac{I_o}{I}\right) = \frac{dI}{I_o}(1 + Kp) \tag{20}$$

where we used Eq. (1) to eliminate I_o/I. I_o is independent of the pressure and is therefore a more appropriate reference quantity in this case. Using Eq. (16), and carrying out some simplifications, we have

$$\hat{\psi}_I \equiv \frac{dC_p}{dI/I_o} = -\frac{(1 + (\gamma - 1)/2M_\infty^2)^{\gamma/(\gamma-1)}}{(\gamma/2M_\infty^2)(Kp_{t\infty})} \times \left(1 + \frac{Kp_{t\infty}}{\{1 + [(\gamma - 1)/2](M/M_\infty)^2 M_\infty^2\}^{\gamma/(\gamma-1)}}\right)^2 \tag{21}$$

where we used the caret in $\hat{\psi}_I$ to signify the fact that I_o is used as a reference intensity.

Figure 3 illustrates the dependence of the influence coefficient on the tunnel stagnation pressure for a freestream Mach number of 1.0, with the three special values of M/M_∞ as parameters. Figure 3 shows that the coefficient $\hat{\psi}_I$ has a minimum near $Kp_{t\infty} = 1$ and rises to high values both for low and high pressures. It can be shown that $Kp = 1$ exactly at the minimum for any combination of the parameters M_∞ and M/M_∞.

The dependence on the local Mach number is straightforward because $\hat{\psi}_I$ decreases monotonically with increasing M/M_∞, and the three selected values of M/M_∞ (0, 1, 2) illustrate the trends of $\hat{\psi}_I$ quite well. The highest values occur at $M/M_\infty = 0$, i.e., at stagnation points or lines. The stagnation points are thus the most critical locations on the model for any given wind tunnel condition. If we set $M = 0$ (i.e., if we restrict ourselves to stagnation points) then the influence coefficient $\hat{\psi}_I$ has an absolute minimum at $Kp_{t\infty} = 1$ and $M_\infty = \sqrt{2}$. The value of the minimum (for air) is

$$(\hat{\psi}_I)_{\min} = 4\gamma^{\gamma/(\gamma-1)} \cong 9.28 \qquad (22)$$

Equation 22 means that at a stagnation point, 1% uncertainty in light intensity ($dI/I_o = 0.01$) results in a C_p uncertainty of about 0.1 or greater. No improvement of paint properties can change this result. A reduction of dC_p is possible only by improving the precision of the light intensity measurement dI and/or by increasing the luminescence intensity I_o.

The dependence of $\hat{\psi}_I$ on the freestream Mach number is illustrated in Fig. 4, in which Kp is held constant at a value of 3, and M_∞ is varied. The parameter is again M/M_∞. Figure 4 illustrates that the influence coefficient, and hence the C_p error, sharply increases as the freestream Mach number is reduced. The reason is that at low Mach numbers, the flow-generated pressure differentials become very small compared with the absolute pressure level for any value of M/M_∞. There is a mild increase of $\hat{\psi}_I$ at high Mach numbers.

The rapidly rising uncertainty at low Mach numbers suggests that the utility of the ratio method, and perhaps any other PSP method, may be limited to the moderate-to-high Mach number range.

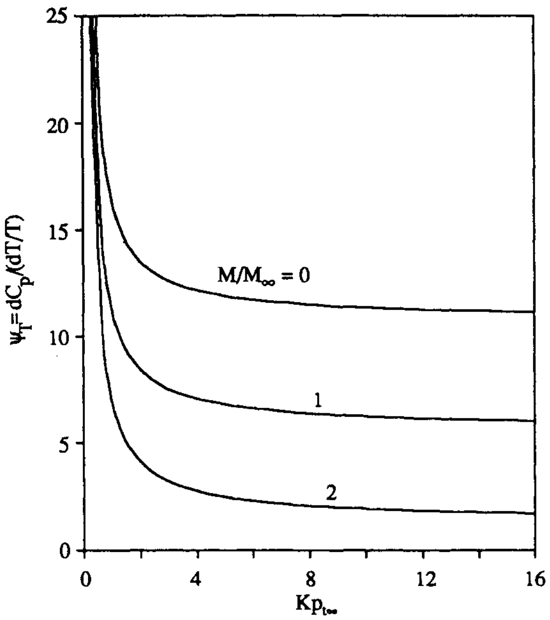

Fig. 5 Influence coefficient ψ_T for temperature at run conditions, as a function of tunnel stagnation pressure: M_∞ = const = 1.0.

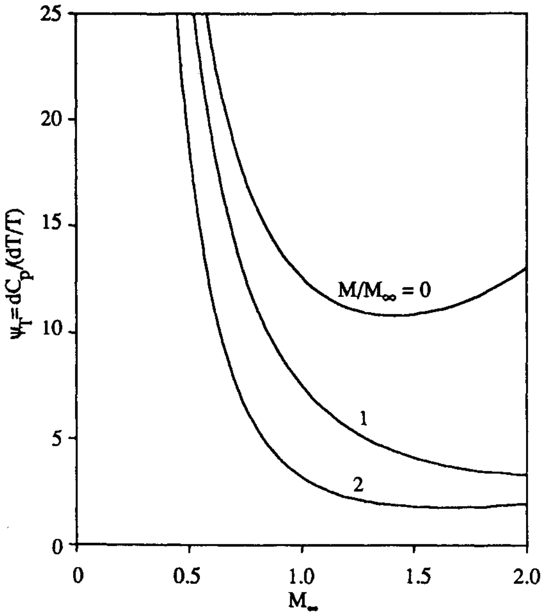

Fig. 6 Influence coefficient ψ_T for temperature at run conditions, as a function of freestream Mach number: $Kp_{t\infty}$ = const = 3.0.

Influence Coefficient for T

Using a procedure analogous to Eqs. (18–21), the following expressions are obtained:

$$\psi_T = \frac{dC_p}{dT/T} = f_1(\nu_Q f_2 - \nu_K) \qquad (23)$$

where

$$f_1 = \frac{2}{\gamma M_\infty^2}\left\{\frac{1 + (\gamma-1)/2M_\infty^2}{1 + [(\gamma-1)/2]\,(M/M_\infty)^2 M_\infty^2}\right\}^{\gamma/(\gamma-1)} \qquad (24a)$$

$$f_2 = 1 + \frac{1}{Kp_{t\infty}}\left[1 + \left(\frac{M}{M_\infty}\right)^2 \frac{\gamma-1}{2} M_\infty^2\right]^{\gamma/(\gamma-1)} \qquad (24b)$$

Figures 5 and 6 illustrate this relation as a function of $Kp_{t\infty}$ and M_∞, respectively, for a paint with $\nu_K = 3.1$ and $\nu_Q = -2.6$. The plots show that ψ_T rises sharply at low pressures, but there is no rise with increasing pressures and there is no minimum. The effect of the freestream Mach number is comparable to what was found for $\hat{\psi}_I$: decreasing Mach numbers mean sharply rising errors. The variation is again monotonic with M/M_∞ as before, stagnation points being the least favorable. The magnitudes of ψ_T are generally high compared to unity, indicating that accurate temperature measurement or some form of temperature compensation is called for.

Summary

The effect of the precision of measuring the primary input quantities on the precision of pressure measurement by the ratio method has been investigated for paints that obey that Stern-Volmer relation. The study shows that the ratio method works best at moderate pressures and moderate-to-high Mach numbers. Precision is diminished at low pressures and at low Mach numbers. For fixed freestream conditions, precision is least at stagnation points or lines and improves with local Mach number.

Depending on the properties of the paint, surface temperature may have a strong influence on the final result. It may be possible to tailor the paint properties in a way to reduce the influence of temperature uncertainties. Otherwise, a precise temperature measurement or compensation method is required, especially if the

PSP system is to be wholly independent of simultaneous, conventional pressure measurements.

Acknowledgments

This work was performed under the McDonnell Douglas Independent Research and Development Program. The author significantly benefitted from and is grateful for numerous stimulating conversations on the subject with Scott D. Schwab, John F. Donovan, and Roger C. Crites.

References

[1]Peterson, J. I., and Fitzgerald, V. F., "New Technique of Surface Flow Visualization Based on Oxygen Quenching of Fluorescence," *Review of Scientific Instruments,* Vol. 51, No. 5, 1980, pp. 670, 671.

[2]"OPMS-S, Optical Pressure Measurement System," brochure by Inteco srl, Via Mola Vecchia 2/A, I-03100 Frosinone, Italy, 1990.

[3]Kavandi, J., Callis, J., Gouterman, M., Khalil, G., Wright, D., Green, E., Burns, D., and McLachlan, B., "Luminescence Barometry in Wind Tunnels," *Review of Scientific Instruments,* Vol. 61, No. 11, 1990, pp. 3340–3347.

[4]Vollan, A., and Alati, L., "A New Optical Pressure Measurement System (OPMS)," *Proceedings of the 14th International Congress on Instrumentation in Aerospace Facilities* (Rockville, MD), Institute of Electrical and Electronics Engineers, Piscataway, NJ, Oct. 1991, pp. 10–16.

[5]Engler, R. H., Hartmann, K., and Schulze, B., "Aerodynamic Assessment of an Optical Pressure Measurement System (OPMS) by Comparison with Conventional Pressure Measurements in a High Speed Wind Tunnel," *Proceedings of the 14th International Congress on Instrumentation in Aerospace Facilities* (Rockville, MD), Institute of Electrical and Electronics Engineers, Piscataway, NJ, Oct. 27–31, 1991, pp. 17–24.

[6]Morris, M. J., Donovan, J. F., Kegelman, J. T., Schwab, S. D., Levy, R. L., and Crites, R. C., "Aerodynamic Applications of Pressure Sensitive Paint," AIAA Paper 92-0264, Jan. 1992.